Trigonometric Identities

Below are the standard Trigonometric Identities you are expected to remember from Precalculus. It would be a good idea to write these down on flash cards and review them every few days until you have them memorized. It is strictly prohibited to have this list copied down in your homework notebook as a list.

1. **Inverse**
$$\csc x = \frac{1}{\sin x};\ \sec x = \frac{1}{\cos x};\ \cot x = \frac{1}{\tan x}$$

2. **Negative Angle**
$$\sin(-\theta) = -\sin\theta;\ \cos(-\theta) = \cos\theta;\ \tan(-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta;\ \sec(-\theta) = \sec\theta;\ \cot(-\theta) = -\cot\theta$$

3. **Pythagorean**
$$\sin^2\theta + \cos^2\theta = 1;\ \tan^2\theta + 1 = \sec^2\theta;\ 1 + \cot^2\theta = \csc^2\theta$$

4. **Sum & Difference**
$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \sin B\cos A \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A\tan B} \\
\tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A\tan B}
\end{aligned}$$

5. **Double Angle**
$$\begin{aligned}
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 1 - 2\sin^2 A \\
&= 2\cos^2 A - 1 \\
\tan 2A &= \frac{2\tan A}{1 - \tan^2 A}
\end{aligned}$$

6. **Half Angle**
$$\begin{aligned}
\sin\frac{A}{2} &= \pm\sqrt{\frac{1 - \cos A}{2}} \\
\cos\frac{A}{2} &= \pm\sqrt{\frac{1 + \cos A}{2}} \\
\tan\frac{A}{2} &= \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}
\end{aligned}$$

7. **Law of Sines and Cosines**
Given a triangle with angle measures α, β, γ and opposite side lengths A, B, C:
$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$
$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

Trigonometry Review

1. Draw a completed unit circle with all intersection points on the axes, three standard angles in each quadrant, and the intersection points for each standard angle labeled.

2. Sketch the graph of sine and cosecant on a single axis; sketch the graph of cosine and secant on a single axis; sketch the graph of tangent and cotangent on a single axis.

3. Explain what each of the coefficients in the below equation do to the standard graph of sine. What is similar and different if it was a cosine or tangent function?

$$f(x) = A\sin\left(B\left(x - C\right)\right) + D$$

4. Graph the below function being sure to label all important points.

$$g(\theta) = 4\cos\left(\frac{1}{2}\left(\theta - \pi/3\right)\right) - 3$$

5. Solve the below trigonometric equations.

   (a) $\cos^2 x = 0.5$ on $0 \leq x \leq 2\pi$

   (b) $\cos 3\theta = \sin 3\theta$ on $0 \leq \theta \leq 2\pi$

6. Although the six trigonometric functions are not one-to-one, their domains can be restricted to make them one-to-one functions and then inverse functions can be defined. Write down the definition, domain, and range of each of the six inverse trigonometric functions. Note, it may be helpful to look at your graphs from **??**. when considering how to restrict the original domain.

7. Simplify the below expression using a right triangle assuming that the angle x has positive measure.

$$\sin\left(\cos\left(x/2\right)\right)$$

8. Evaluate the below expressions without a calculator. If an indeterminate is present, assume it is positive.

   (a) $\csc^{-1}\left(-1\right)$

   (b) $\tan^{-1}\left(\tan\left(3\pi/4\right)\right)$

   (c) $\tan\left(\cos^{-1} x\right)$

   (d) $\cot\left(\tan^{-1} 2\theta\right)$